The Jefferson Performing Arts Society

Presents

Sister Act
A Divine Musical Comedy

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Sister Act is the feel-amazing musical comedy smash based on the hit 1992 film that has audiences jumping to their feet! This uplifting musical was nominated for 5 TONY Awards including Best Musical.

This JPAS Study Companion provides students with opportunities to explore Sister Act within the context of Greek Tragedy, physics and mathematics. Students are first introduced to Sister Act with a brief history of the show and information on the artists who helped create it. Next, Violence On and Off Stage: A Brief History gives students opportunities to learn about the origins of theatre, particularly Greek Tragedy and the convention of depicting violence off stage. Stained Glass and Four Color Theory gives students opportunities to consider the math behind the design theory, and then create their own. The Golden Ratio in Shapes and Sounds expands on design theory and gives students additional opportunities to consider math in nature and how these same mathematical patterns can be found in stained glass designs.
and sound waves. Students also have opportunities to create their own designs. Additionally, this section provides scientific information about the human eye, sight, and how the Golden Mean can influence how we see. **Raise your Voice: Translating Song into Mathematics** gives students the opportunity to raise their own voices, learn about the physics behind resonance, and compare the sound of their voices with the rest of the class.

*The first thing I wanted to do was think of the nuns like the townspeople in The Music Man. I wanted there to be lots of little stories going on with them, like about the nun who is maybe a little bit out of it, so I got to throw a few more characters into the mix.* ~~ Douglas Carter Beane

ALL:
Get down!
Get down!
With all your heart and your soul!
Dance on!
Dance on! (wooo)
Become a part of the whole!
Don’t stop!
Don’t stop!
Until your lost in the sound!
Life’s only love!
Spread the love! (spread it)
Spread the love! (spread it)
Spread the love! (spread it)
Spread the love…love…love AROUND!

Louisiana Educational Content Standards and Benchmarks

The arts facilitate interconnection. They provide tangible, concrete opportunities for students and teachers to explore academic concepts. The arts are even more critical now with the introduction of Louisiana Common Core. Common Core is replacing the system of Grade Level Expectations and Standards and Benchmarks previously used to measure student achievement. Here is some background information on Louisiana Common Core:

COMMON CORE STATE STANDARDS
Academic standards define the knowledge and skills that students are expected to learn in a subject in each grade. In 2010, Louisiana adopted Common Core State Standards in English language arts and math. The Common Core State Standards define what students need to learn in reading, writing and math in each grade to stay on track for college and careers. Please visit this site for more information: http://www.louisianabelieves.com/academics/louisiana-student-standards-review

All Common Core connections were retrieved from:


Background
Sister Act

Music by Alan Menken
Lyrics by Glenn Slater
Book by Cheri Steinkellner and Bill Steinkellner
Additional Book Material by Douglas Carter Beane
Based on the Touchstone Pictures Motion Picture Sister Act written by Joseph Howard

2 Acts, Book Musical, Rated PG

Sister Act is currently restricted for licensing, click here to be one of the first people to receive a special email message notifying you as soon as this show becomes available for licensing. U.S. National Tour Version (2012)

A woman hiding in a convent helps her fellow sisters find their voices as she rediscovers her own.

Sister Act is the feel-amazing musical comedy smash based on the hit 1992 film that has audiences jumping to their feet! Featuring original music by TONY and 8-time Oscar winner Alan Menken (Newsies, Beauty And The Beast, Little Shop Of Horrors), this uplifting musical was nominated for 5 TONY Awards including Best Musical.

When disco diva Deloris Van Cartier witnesses a murder, she is put in protective custody in one place the cops are sure she won't be found: a convent! Disguised as a nun, she finds herself at odds with both the rigid lifestyle and uptight Mother Superior. Using her unique disco moves and singing talent to inspire the choir, Deloris breathes new life into the church and community but in doing so, blows her cover. Soon, the gang is giving chase only to find them up against Deloris and the power of her newly found sisterhood.

Filled with powerful gospel music, outrageous dancing and a truly moving story, Sister Act will leave audiences breathless. The cast is chock full of amazing roles for women of all ages making this a perfect choice for high schools and community theatres. A sparkling tribute to the universal power of friendship, Sister Act is reason to REJOICE!

Synopsis

ACT ONE

It is Christmas Eve in 1977 at a disco nightclub in Philadelphia. Club singer Deloris Van Cartier, and her two back-ups, are performing in front of an empty house. A gangster and four henchmen then appear. Van Cartier's boyfriend, Curtis Jackson, owns the nightclub (“Take Me To Heaven”). Deloris is overjoyed as she believes her boyfriend is going to introduce her to a record producer on that day. She soon learns, however, that
this is not to be. Jackson tells her he cannot join her for Christmas Day but gives her a coat, which she discovers belonged to his wife. Hurt and frustrated, Deloris goes to her backup singers Michelle and Tina about her dreams of stardom and fame ("Fabulous, Baby!"). She decides to break up with Jackson and head out of Philadelphia to go fulfill her dreams on her own. When she happens to witness Jackson and his crew murder someone accused of 'squealing' to the cops, however, she becomes terrified. Deloris promptly runs away as Jackson orders his men to get her and bring her back.

Some time later, Deloris runs into a police station and tells the desk chief, Eddie, about what has happened. The two recognize each other as old friends from school with Deloris calling him Sweaty Eddie. Eddie decides that Deloris needs to go into the witness protection program and sends her to the place he believes Jackson will never find her...a convent. When Deloris arrives, she is disappointed as she learns from the Mother Superior that contact with the outside world is limited and that she cannot smoke, drink, or wear any of her less than appropriate clothing ("Here Within These Walls").

Deloris joins the other nuns for dinner, is introduced as Sister Mary Clarence, and after several comedic interactions with the overly perky Sister Mary Patrick, Deloris discovers just how much is limited when she is a nun ("It's Good To Be A Nun"). Deloris then asks them about what they're missing, and Mother Superior makes Deloris go on a fast. Meanwhile back in his nightclub, Jackson is frustrated that he cannot find Deloris anywhere. He tells Joey, TJ, and Pablo how he will not stop until he finds and kills Deloris ("When I Find My Baby").

Back at the convent, Deloris is hungry from the fast. She goes across the street to a slinky bar and is followed by Sister Mary Patrick and Sister Mary Robert, a shy and quiet nun. When the two nuns arrive they see how fun things can be outside of the convent. However, Deloris recognizes Joey, TJ, and Pablo entering the bar. Suddenly there is a fight in the bar, giving Deloris, Sister Mary Patrick, and Sister Mary Robert a chance to escape. Eddie and Mother Superior meet them back at the convent. Mother Superior confronts Deloris, telling her that she must conform to the life of the nuns. Eddie agrees and reveals that Jackson has upped the price on her head, so she needs to be careful. Deloris storms back to the convent after being informed that she has to wake up at 5 a.m. and join the choir. Eddie, now alone with only the drunks and homeless on the street, privately reflects on his desire to just let go and impress Deloris ("I Could Be That Guy").

The following morning, Deloris attends the choir practice and realizes how bad the situation is. Deloris offers to teach them thus taking over from Sister Mary Lazarus, an older nun. Deloris teaches the nuns how to sing in key and on time. She also manages to break the quiet and timid Sister Mary Robert out of her shell ("Raise Your Voice"). That Sunday, the choir performs a rousing up-tempo hymn ("Take Me To Heaven [Reprise]"). Although the church is struggling, the improved performance and new material bring an influx of donations and membership. Mother Superior, however, is horrified how the simple traditional choir she knew has changed and become modern.
ACT TWO

Mother Superior wants to get rid of Deloris, but Monsignor O'Hara reminds her that the men who were planning to buy the church love the choir and have given their money to improve and keep the church because of the choir. Over the last few weeks, the choir has become incredibly successful and the money from donations has paid for the church to be remodeled and fixed ("Sunday Morning Fever").

Later, Monsignor O'Hara bursts in with exciting news: the choir has been asked to perform a special concert in front of the Pope. Deloris is thrilled but also regretful; Jackson and his goons have spotted Deloris with the choir on the television accepting the invitation. Jackson orders his henchmen to get into the convent and bring Deloris back. TJ, Joey, and Pablo discuss how they will do this ("Lady In The Long Black Dress"). Meanwhile, Mother Superior prays to God and asks why He has given her the challenge that is named Deloris ("I Haven't Got A Prayer"). She soon receives a call from Eddie.

Deloris is approached by the Nuns before they go to sleep, asking her to lead them praying for their show for the Pope, which is the following day ("Bless The Show"). Mother Superior arrives shortly thereafter and informs Deloris that the court date for Jackson has moved up to the next day and she must leave immediately. The other nuns overhear and Deloris is forced to tell them the truth about who she really is and that she cannot perform with them. Deloris quickly runs off to get her things followed by Sister Mary Robert while the other nuns disappointingly go back to their rooms.

As Deloris prepares to leave, Sister Mary Robert begs to come with her claiming that she has been inspired to become a stronger person and go after the things she wants ("The Life I Never Led"). Deloris tells her that she doesn't need her to do that, and that she can do it all herself if she really wants to. Deloris runs from the convent and stays at Eddie's house for the night. While there she initially is overjoyed that the following day she will testify against Jackson and his boys to go back to pursuing the career she's dreamed of ("Fabulous, Baby! [Reprise]"). Feeling immense guilt for abandoning her sisters when they needed her, however, she begins to reflect on her life and finally realizes that the choices she is making will leave her with nothing but fame and money. She decides to return to the convent and sing with her sisters ("Sister Act").

Elsewhere, Jackson thinks up a new way to get into the convent ("When I Find My Baby [Reprise]"). Dressed as nuns, Jackson, Joey, TJ, and Pablo sneak into the convent. They find and chase Deloris. The Nuns see that, and they all prepare to protect Deloris. Mother Superior, however, is adamant that they stay away and call the police, which prompts an outburst from Sister Mary Robert who tells her that she won't be quiet anymore ("The Life I Never Led [Reprise]"). The other nuns agree and all go running through the convent looking for Deloris.

Deloris eventually runs into Jackson, leading to a final confrontation. He is armed and
dangerous, and she is scared but calm. Jackson demands that Deloris to get on her knees and beg for her life. However, all the sisters and nuns run in unafraid; they stand in front of her telling Jackson that they will have to go through them first ("Sister Act [Reprise]"). Jackson is about to start firing at the nuns when Eddie comes in and fights Jackson off. He arrests Jackson and his boys. He and Deloris then share a passionate embrace. Mother Superior and Deloris come to a truce and agree to accept each other - - warts and all. As the curtain begins to fall, we transition to the performance in front of the Pope as the nuns stand in unison with their voices ringing in harmony, led by a changed Deloris Van Cartier ("Spread The Love Around").

RETRIEVED FROM: http://www.mtishows.com/show_detail.asp?showid=000431
Show History

Inspiration

*Sister Act* is based on the hit 1992 comedy film of the same name, originally starring Whoopi Goldberg. With a movie so popular and musically focused, Stage Entertainment decided to produce a stage adaptation. They brought on Oscar and Tony-winning composer Alan Menken, along with lyricist and frequent collaborator Glenn Slater. Bill and Cheri Steinkellner were hired to write the book.

Menken wanted to inspire his score off of 1970s music, specifically disco and gospel. As a result, the setting was changed from Reno and San Francisco in the 90s (the original setting of the movie) to Philadelphia in the 70s. In the transfer from the West End to Broadway, not only did Whoopi Godlberg herself jump on as a producer, but the script underwent several revisions. Douglas Carter Beane kept the structure of the show and made several large changes.

Productions

*Sister Act*, based on the hit film of the same name, premiered at the Pasadena Playhouse in California running from October 24 to December 23, 2006. While in residence, it broke various box office records. The production ended up moving to the Alliance Theatre in Atlanta, Georgia and ran from January 17 to February 25, 2007. The musical then moved across the pond. The West End premiere was on June 2, 2009 at the London Palladium, starring Patina Miller, Sheila Hancock, and Ian Lavender. It ran for more than a year and closed on October 30, 2010.

*Sister Act* opened on Broadway at the Broadway Theatre on April 20, 2011. After the West End production closed, the show went through a multitude of revisions, helmed by book writer Douglas Carter Beane. Patina Miller transferred with the production in the role of Deloris, costarring with Victoria Clark, Fred Applegate, and Chester Gregory. It closed on August 26, 2012 after 561 performances.

The musical has launched two tours: one in the UK and Ireland that ran from October 2010 to October 2012, and one in North America that opened during the fall of 2012. *Sister Act* has also been performed in quite a few countries around the world, including Germany, Italy, the Netherlands, Australia, South Korea, and Mexico.

Critical Reaction

"One of the season’s happiest surprises [...] Menken evokes the lush, funky sound of Philly soul without falling into mere pastiche." -The NY Post
"Composer Alan Menken and lyricist Glenn Slater provide original tunes that nod cheekily, but with genuine affection, to that pop era while also propelling the story with a style and exuberance specific to well-crafted musical theater. Librettists Cheri and Bill Steinkellner, enlisting additional material from Douglas Carter Beane, adapt the screenplay with disarming wryness." -USA Today

"When the wimples start quivering, the pinched mouths break into sunbeam smiles, and the nuns start rocking to raise the Gothic rafters, all’s right in the kingdom of musical comedy." -The New York Times

RETRIEVED FROM: http://www.mtishows.com/show_detail.asp?showid=000431
About the Author:
As the musical adaptation of *Sister Act* was preparing to hop the pond from the West End to Broadway, producers enlisted a veteran playwright to give the show a Broadway makeover: Douglas Carter Beane. The famously funny writer earned his third Tony Award nomination for his *Sister Act* efforts, having gotten nods for his two previous Rialto outings, the satirical play *The Little Dog Laughed* and tongue-in-cheek book for the musical *Xanadu*. Here, Beane reflects on the challenges of revamping a show’s existing script and shares his wisdom on what really makes “a Broadway show.”

There is such a thing as “a Broadway show,” and *Sister Act* needed to be a Broadway show. There’s a muscularity to it, there’s a pace to it, the characters are a certain way, and I just knew a New York audience would really go for and understand this piece. But it wasn’t there yet. As fate would have it, I happen to have grown up in Philadelphia in the ’70s, when the drinking age was like a suggestion: You had to be 18 years old or very tall to get a cocktail. It was a different time! So even though I was incredibly underage, I was 6’4” and could get into clubs and I saw those amazing people that Deloris Van Cartier so badly wants to be.

From the beginning, the work was a constant spiral of drafts. As I was re-writing the first scenes I was writing the next scene so my typist, Paul Downs Colaizzo, became my assistant writer and by the end was like an associate writer. And he was a Catholic, which helped. I would sit there and go, “What’s a novena? What are the stations of the cross?” and he would shout it out. The biggest challenge was structuring a compelling story around the existing song order. Glenn [Slater] and Alan [Menken] were very committed to the song structure as it existed in the London version, and I totally understood why, so it was about writing around it while getting
in the mojo and the fun and the characters. The first thing I wanted to do was think of the nuns like the townspeople in *The Music Man*. I wanted there to be lots of little stories going on with them, like about the nun who is maybe a little bit out of it, so I got to throw a few more characters into the mix.

CREATIVE TEAM

LAN MENKEN - COMPOSER

Has garnered 8 Oscars, 12 Grammys and a Tony for his scores, including Little Shop of Horrors, The Little Mermaid, Beauty and the Beast, Aladdin, A Christmas Carol, Newsies, Pocahontas, The Hunchback of Notre Dame, Hercules, Enchanted, Tangled, Sister Act, Leap Of Faith, God Bless You, Mr. Rosewater, King David and The Apprenticeship of Duddy Kravitz. Inducted into the Songwriter’s Hall of Fame, Doctorates from New York University and North Carolina School of the Arts.

GLENN SLATER - LYRICS

Tangled (2011 Grammy Winner, 2010 Oscar and Golden Globe nominee); Sister Act (2011 Tony nominee—Best Score); Andrew Lloyd Webber’s Love Never Dies (2010 Olivier nominee—Best Musical); Disney’s The Little Mermaid (2008 Tony nominee—Best Score, Grammy nominee—Best Cast Album); Leap of Faith (Tony nominee—Best Musical, 2012); Disney’s Home On The Range (2004). Glenn lives in New York City with composer/lyricist/wife Wendy Leigh Wilf and sons Benjamin and Daniel.

CHERI & BILL STEINKELLNER - BOOK

Multiple Emmys, Golden Globes, Peoples’ Choice, BAFTA, Writers’ Guild, and TV Land Legend Awards for writing/producing Cheers and Teacher’s Pet. 2011 Tony-nominees for Sister Act. Bill co-wrote The PeeWee Herman Show, Cheri the award-winning Hello! My Baby, soon to be California’s first All-State Musical. Founders of Instaplay - L.A.’s original all-improvised musical, the Steinkellners teach at Stanford, USC and UCSB in Santa Barbara where they raised their favorite children/writers/artists: Kit, Teddy and Emma.
DOUGLAS CARTER BEANE - ADDITIONAL BOOK MATERIAL

Broadway: Rodgers & Hammerstein's Cinderella, The Nance, Lysistrata Jones (Tony nom.), Sister Act (Tony nom.), Xanadu (Tony nom.), The Little Dog Laughed (Tony nom., GLAAD Media Award) West End: The Little Dog Laughed (Olivier nom.) Off-Broadway: As Bees in Honey Drown (Outer Critic's Circle Award), Mr. & Mrs. Fitch, Music from a Sparkling Planet, The Cartells, Mondo Drama, The Country Club, Advice From a Caterpillar. Doug thanks Paul Rudnick for the original idea and Paul Downs Colaizzo for his assistance.

RETRIEVED FROM: http://sisteractontour.com/creative.html
Violence On and Off Stage: A Brief History
In Act One, Deloris is an eye witness to murder. Jackson and his crew murder someone accused of 'squealing' to the cops. Deloris promptly runs away as Jackson orders his men to get her and bring her back. Rather depict the violence and show the audience what Deloris sees, JPAS *Sister Act* Director Kris Shaw chooses to borrow a convention from Greek theatre. Within Greek Tragedy, As a consequence of the serious subject matter, which often dealt with moral right and wrongs, no violence was permitted on the stage. The death of a character had to be heard from offstage and not seen.

Additionally, Classical Greek drama would use the chorus to describe to the audience what had occurred off stage. In Classical Greek drama the chorus was a group of actors who described and commented upon the main action of a play with song, dance, and recitation. In the case of the JPAS production of *Sister Act*, violence occurs off stage and then Deloris describes what has happened.
The Tangled Ways of Zeus: And Other Studies In and Around Greek Tragedy

Alan H. Sommerstein

Print publication date: 2010
Published to Oxford Scholarship Online: September 2010
DOI: 10.1093/acprof:oso/9780199568314.001.0001

A brief overview to identify and explain the constraints on the onstage presentation of violence in Greek drama: there were two conventions: that the audience must never witness any act or occurrence that impinged on a human or animal body so as to be the proximate cause of a death, and that in tragedy, the audience must not witness any person inflicting a blow on any other person. The first convention was based on religious considerations, and was unbreakable; the second was based on artistic considerations (or perhaps just on generic tradition), and is broken in one play, Prometheus Bound, for the special purpose of emphasizing the unimaginable agonies that Prometheus is suffering.

RETRIEVED FROM:
impinge

Also found in: Medical, Legal, Idioms, Encyclopedia, Wikipedia.

im·pinge

(im-pīnj’)

v. im·pinged, im·ping·ing, im·ping·es

v.intr.

1.

a. To encroach on or limit something, such as a right: “powerful institutions of government that inhibited free enterprise and impinged on commercial—and by extension private—liberties” (Greg Critser).

b. Usage Problem To have an effect or influence: “Any consequence of a change in alleles ... is fair game for natural selection, so long as it impinges on the survival of the responsible allele, relative to its rivals” (Richard Dawkins).

2.

a. To collide or strike against something: Sound waves impinge on the eardrum.

b. To advance over or press upon something: pain caused by a bone impinging upon a nerve.

v.tr.

To encroach upon; limit: “One of a democratic government’s continuing challenges is finding a way to protect ... secrets without impinging the liberties that democracy exists to protect” (Christian Science Monitor).

[Latin impingere: in-, against; see in-² + pangere, to fasten; see pag- in Indo-European roots.]

im·pinge′ment n.

im·ping’er n.

Usage Note: The use of impinge meaning “to encroach; trespass,” as in Americans dislike any policy that impinges on their liberty, is well-established as standard. However, when impinge is used more loosely to mean “to have an effect” the Usage Panel is split. In our 2001 survey, only 47 percent of the Panel found t
he following sentence to be acceptable: *What the recovered diary revealed about the villagers directly impinged on the lives of people living there many years later.*


**impinge**

(ɪmˈpɪndʒ)  

*vb*

1. (*intr*; usually foll by *on* or *upon*) to encroach or infringe; trespass: *to impinge on someone's time.*

2. (*intr*; usually foll by *on*, *against*, or *upon*) to collide (with); strike

[C16: from Latin *impingere* to drive at, dash against, from *pangere* to fasten, drive in]

**imˈpingement n**  

**imˈpinger n**


**im′pinge**

(ɪmˈpɪndʒ)  

*v.i.* -pinged, -ping·ing.

1. to encroach; infringe: *to impinge on another's rights.*

2. to strike; collide: *light impinging on the lens.*

3. to make an impression; have an effect: *ideas that impinge upon the imagination.*

[1525–35; < Medieval Latin *impingere* to cause to collide, force = Latin *im-*¹ + - *pingere*, comb. form of *pangere* to fasten, drive in, fix; compare *impact*]

**im′pinge′ment, n.**


**impinge**

, *infringe* -  

To impinge is to come into contact or encroach or have an impact; to infringe is to encroach on a right or privilege or toviolate.

Greek Tragedy

Definition

by Mark Cartwright
published on 16 March 2013

Greek tragedy was a popular and influential form of drama performed in theatres across ancient Greece from the late 6th century BCE. The most famous playwrights of the genre were
Aeschylus, Sophocles, and Euripides and many of their works were still performed centuries after their initial premiere. Greek tragedy led to Greek comedy and, together, these genres formed the foundation upon which all modern theatre is based.

THE ORIGINS OF TRAGEDY

The exact origins of tragedy (tragōida) are debated amongst scholars. Some have linked the rise of the genre, which began in Athens, to the earlier art form, the lyrical performance of epic poetry. Others suggest a strong link with the rituals performed in the worship of Dionysos such as the sacrifice of goats - a song ritual called tragōdia - and the wearing of masks. Indeed, Dionysos became known as the god of theatre and perhaps there is another connection - the drinking rites which resulted in the worshipper losing full control of their emotions and in effect becoming another person, much as actors (hupokritai) hope to do when performing. The music and dance of Dionysiac ritual was most evident in the role of the chorus and the music provided by an aulos player, but rhythmic elements were also preserved in the use of first, trochaic tetrameter and then iambic trimeter in the delivery of the spoken words.

A TRAGEDY PLAY

Performed in an open-air theatre (theatron) such as that of Dionysos in Athens and seemingly open to all of the male populace (the presence of women is contested), the plot of a tragedy was almost always inspired by episodes from Greek mythology, which we must remember were often a part of Greek religion. As a consequence of this serious subject matter, which often dealt with moral right and wrongs, no violence was permitted on the stage and the death of a character had to be heard from offstage and not seen. Similarly, at least in the early stages of the genre, the poet could not make comments or political statements through the play, and the more
The direct treatment of contemporary events had to wait for the arrival of the less austere and conventional genre, Greek comedy.

The early tragedies had only one actor who would perform in costume and wear a mask, allowing him the presumption of impersonating a god. Here we can see perhaps the link to earlier religious ritual where proceedings might have been carried out by a priest. Later, the actor would often speak to the leader of the chorus, a group of up to 15 actors who sang and danced but did not speak. This innovation is credited to Thespis in c. 520 BCE. The actor also changed costumes during the performance (using a small tent behind the stage, theskēne, which would later develop into a monumental façade) and so break the play into distinct episodes. Phrynichos is credited with the idea of splitting the chorus into different groups to represent men, women, elders, etc. (although all actors on the stage were in fact male). Eventually, three actors were permitted on stage - a limitation which allowed for equality between poets in competition. However, a play could have as many non-speaking performers as required, so, no doubt, plays with greater financial backing could put on a more spectacular production with finer costumes and sets. Finally, Agathon is credited with adding musical interludes unconnected with the story itself.

TRAGEDY IN COMPETITION

**BESIDES PERFORMANCE IN COMPETITION, MANY PLAYS WERE COPIED INTO SCRIPTS FOR PUBLICATION AND POSTERITY.**

The most famous competition for the performance of tragedy was as part of the spring festival of Dionysos Eleuthereus or the City Dionysia in Athens, but there were many others. Those plays which sought to be performed in the competitions of a religious festival (agōn) had to go through an audition process judged by the archon. Only those deemed worthy of the festival would be given the financial backing
necessary to procure a costly chorus and rehearsal time. The *archon* would also nominate the three *chorēgoi*, the citizens who would each be expected to fund the chorus for one of the chosen plays (the state paid the poet and lead actors). The plays of the three selected poets were judged on the day by a panel and the prize for the winner of such competitions, besides honour and prestige, was often a *bronze* tripod cauldron. From 449 BCE there were also prizes for the leading actors (*prōtagōnistēs*).

A bust of *Sophocles* (497/6 - 406 BCE) the *Greek* tragic poet and author of such masterpieces of *Greek Tragedy* as 'Oedipus the King'. Second half of the 1st century BCE. (Vatican Museums, *Rome*)

**THE WRITERS OF TRAGEDY**

The first of the great tragedian poets was Aeschylus (c. 525 - c. 456 BCE). Innovative, he added a second actor for minor parts and by including more dialogue into his plays, he squeezed more drama from
the age-old stories so familiar to his audience. As plays were submitted for competition in groups of four (three tragedies and a satyr-play), Aeschylus often carried on a theme between plays, creating sequels. One such trilogy is *Agamemnon*, *The Libation Bearers* (or *Cheoephoroi*), and *The Furies* (or *Eumenides*) known collectively as the *Oresteia*. Aeschylus is said to have described his work, consisting of at least 70 plays of which six or seven survive, as ‘morsels from the feast of Homer’ (Burn 206).

The second great poet of the genre was Sophocles (c. 496-406 BCE). Tremendously popular, he added a third actor to the proceedings and employed painted scenery, sometimes even changes of scenery within the play. Three actors now permitted much more sophistication in terms of plot. One of his most famous works is *Antigone* (c. 442 BCE) in which the lead character pays the ultimate price for burying her brother Polynices against the wishes of King Kreon of *Thebes*. It is a classic situation of tragedy - the political right of having the traitor Polynices denied burial rites is contrasted against the moral right of a sister seeking to lay to rest her brother. Other works include *Oedipus the King* and *The Women of Trāchis*, but he in fact wrote more than 100 plays, of which seven survive.

The last of the classic tragedy poets was Euripides (c. 484-407 BCE), known for his clever dialogues, fine choral lyrics and a certain realism in his text and stage presentation. He liked to pose awkward questions and unsettle the audience with his thought-provoking treatment of common themes. This is probably why, although he was popular with the public, he won only a few festival competitions. Of around 90 plays, 19 survive, amongst the most famous being *Medeia* - where Jason, of the Golden Fleece fame, abandons the title character for the daughter of the King of *Corinth* with the consequence that Medeia kills her own children in revenge.
THE LEGACY OF TRAGEDY

Although plays were specifically commissioned for competition during religious and other types of festivals, many were re-performed and copied into scripts for ‘mass’ publication. Those scripts regarded as classics, particularly by the three great Tragedians, were even kept by the state as official and unalterable state documents. Also, the study of the ‘classic’ plays became an important part of the school curriculum.

There were, however, new plays continuously being written and performed, and with the formation of actors’ guilds in the 3rd century BCE and the mobility of professional troupes, the genre continued to spread across the Greek world with theatres becoming a common feature of the urban landscape from Magna Graecia to Asia Minor.

In the Roman world, tragedy plays were translated and imitated in Latin, and the genre gave rise to a new art form from the 1st century BCE, pantomime, which drew inspiration from the presentation and subject matter of Greek tragedy.
BIBLIOGRAPHY


ABOUT THE AUTHOR

MARK CARTWRIGHT

Mark holds an M.A. in Greek philosophy and his special interests include the Minoans, the ancient Americas, and world mythology. He loves visiting and reading about historic sites and transforming that experience into free articles accessible to all.

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RETRIEVED FROM: http://www.ancient.eu/Greek_Tragedy/
Chorus

in drama and music, those who perform vocally in a group as opposed to those who perform singly. The chorus in Classical Greek drama was a group of actors who described and commented upon the main action of a play with song, dance, and recitation. Greek tragedy had its beginnings in choral performances, in which a group of 50 men danced and sang dithyrambs—lyric hymns in praise of the god Dionysus. In the middle of the 6th century BC, the poet Thespis reputedly became the first true actor when he engaged in dialogue with the chorus leader. Choral performances continued to dominate the early plays until the time of Aeschylus (5th century BC), who added a second actor and reduced the chorus from 50 to 12 performers. Sophocles, who added a third actor, increased the chorus to 15 but reduced it to a mainly commentarial role in most of his plays (for an example of this role as shown in the play Oedipus the King, see video). The chorus in Greek comedy numbered 24, and its function was displaced eventually by interspersed songs. The distinction between the passivity of the chorus and the activity of the actors is central to the artistry of the Greek tragedies. While the tragic protagonists act out their defiance of the limits subscribed by the gods for man, the chorus expresses the fears, hopes, and judgment of the polity, the average citizens. Their judgment is the verdict of history.

As the importance of the actors increased, the choral odes became fewer in number and tended to have less importance in the plot, until at last they became mere decorative interludes separating the acts. During the Renaissance the role of the chorus was revised. In the drama of Elizabethan England, for instance, the name chorus
designated a single person, often the speaker of the prologue and epilogue, as in Christopher Marlowe’s *Doctor Faustus*.

The use of the group chorus has been revived in a number of modern plays, such as Eugene O’Neill’s *Mourning Becomes Electra* (1931) and T.S. Eliot’s *Murder in the Cathedral* (1935).

In music, chorus refers to the organized body of singers in opera, oratorio, cantata, and church music; to compositions sung by such bodies; to the refrain of a song, sung by a group of singers, between verses for solo voice; and, as a medieval Latin term, to the crwth (the bowed lyre of medieval Wales) and to the bagpipe. (See choir.)

In musicals, the chorus, a group of players whose song and dance routines usually reflect and enhance the development of the plot, became increasingly more prominent during the 20th century. During the late Victorian era, musical comedy was characterized by thin plot, characters, and setting, the main attraction being the song and dance routines, comedy, and a line of scantily clad chorus girls. Their performances provided an extravagant bonus at the beginnings and ends of songs or special dance numbers, and they were considered the flashy sex symbols of the day. As musicals developed, however, more attention was given to integrating their various elements. In the mid-1920s, song and dance numbers began to stem more naturally from the plot, and the chorus danced more than it sang. The dancing itself soon developed from the lines of synchronized leg kicking of the early 1900s into highly sophisticated ballet and modern dance.

RETRIEVED FROM: http://www.britannica.com/art/chorus-theatre
Stained Glass and Set Design
JPAS Sister Act:

Stained Glass and Four Color Theory

By Karel Sloane-Boekbinder

The set for Sister Act incorporates backdrops painted with stained glass designs. This lesson gives students opportunities to consider the math behind the design theory, and then create their own.

Begin by viewing imagery of JPAS stained glass backdrops. Display the images where everyone can see them, such as a dry-erase board, Promethean board or ELMO. Ask students to identify the shapes they see (polygons, triangles, rectangles, quadrilaterals, parallelograms.) As a class, discuss the shapes.

Here are some examples: A polygon is a plane shape with straight sides. Polygons are 2-dimensional shapes. They are made of straight lines, and the shape is "closed" (all the lines connect up); quadrilateral means "four sides" (quad means four, lateral means side). A Quadrilateral has four-sides, it is 2-dimensional (a flat shape), closed (the lines join up), and has straight sides. Use the Names of Polygons table as a reference sheet; display the reference sheet where everyone can see it.

Next, describe how these shapes are made into patterns and how color is used within the patterns. Introduce information about The Four Color Theorem. Ask students to look at imagery of JPAS stained glass backdrops and discuss how The Four Color Theorem is used to color in the shapes.

Distribute the JPAS stained glass backdrops color sheets. Distribute markers or crayons. Ask students to use The Four Color Theorem to create their own designs.
JPAS Stained Glass Back Drop: Image 2
Names of Polygons

<table>
<thead>
<tr>
<th>Name</th>
<th>Sides</th>
<th>Shape</th>
<th>Interior Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle (or Trigon)</td>
<td>3</td>
<td><img src="triangle.png" alt="Triangle" /></td>
<td>60°</td>
</tr>
<tr>
<td>Quadrilateral (or Tetragon)</td>
<td>4</td>
<td><img src="quadrilateral.png" alt="Quadrilateral" /></td>
<td>90°</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td><img src="pentagon.png" alt="Pentagon" /></td>
<td>108°</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td><img src="hexagon.png" alt="Hexagon" /></td>
<td>120°</td>
</tr>
<tr>
<td>Heptagon (or Septagon)</td>
<td>7</td>
<td><img src="heptagon.png" alt="Heptagon" /></td>
<td>128.571°</td>
</tr>
<tr>
<td>Octagon</td>
<td>8</td>
<td><img src="octagon.png" alt="Octagon" /></td>
<td>135°</td>
</tr>
</tbody>
</table>
Coloring (The Four Color Theorem)

This activity is about coloring, but don't think it's just kid's stuff. This investigation will lead to one of the most famous theorems of mathematics and some very interesting results.

Have you ever colored in a pattern and wondered how many colors you need to use?

There is only one rule

**Two sections that share a common edge cannot be colored the same!**

Having a common corner is OK, just not an edge.

Let's start with a simple pattern like a group of nine squares:
How many colors do you need to color the pattern of nine squares?

You could use nine different colors, but could make do with as few as two:

A Little More Complicated

How about this one?
How many colors do you need this time?

Your turn ... try it ... then scroll down to see my answer

...  

...  

You could use four different colors, or you could make do with just three:  

But you couldn't color this pattern with just two colors. Can you see why?
Stained Glass and Four Color Theory

JPAS stained glass backdrop color sheet 1
Stained Glass and Four Color Theory

JPAS stained glass backdrop color sheet 2
Definition of Quadrilateral RETRIEVED FROM:
https://www.mathsisfun.com/quadrilaterals.html

Definition of polygon RETRIEVED FROM:
https://www.mathsisfun.com/geometry/polygons.html

Information about The Four Color Theory RETRIEVED FROM:
http://www.mathsisfun.com/activity/coloring.html
Mathematics Standards » Literacy Standards »

Grade 4

Operations and Algebraic Thinking 4.OA

A. Use the four operations with whole numbers to solve problems.

C. Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint...

Geometry 4.G

A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines.

Identify these in two-dimensional figures.

Mathematics » Grade 7

Geometry 7.G

A. Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

B. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
Mathematics Standards » Algebra II

Linear, Quadratic, and Exponential Models★ F-LE

A. Construct and compare linear, quadratic, and exponential models and solve problems.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

B. Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.
JPAS Sister Act:

The Golden Ratio in Shapes and Sounds

By Karel Sloane-Boekbinder

The previous lesson explored how the set for Sister Act incorporates backdrops painted with stained glass designs. This lesson expands on design theory and gives students additional opportunities to consider math in nature and how these same mathematical patterns can be found in stained glass designs and sound waves. Students also have opportunities to create their own designs.

Begin by viewing, as a class, the following three images:

- a rose window in a cathedral
- patterns sound waves make in water
- a close up of a human iris

Do not tell students what they are looking at. Display the images where everyone can see them, such as a dry erase board, Promethean board or ELMO. Ask students to consider what these three images have in common; and how they are different. Distribute the “Compare and Contrast Three Images” student worksheet. Ask students to complete the worksheet while viewing the images.
The Golden Ratio in Shapes and Sounds

Compare and Contrast the Three Images
Next, distribute the “Golden Ratio Overview” information sheets. As a class, read and discuss the sheets. During this discussion, reveal the types of images the students have been viewing: a rose window in a cathedral; patterns sound waves make in water; and a close up of a human iris. Distribute the “Compare and Contrast” Essay organizers. Ask students to use these organizers to develop essays that compare and contrast a rose window in a cathedral, patterns sound waves make in water and a human iris and how these three images reflect the “Golden Ratio.”

Give students an opportunity to further reflect on these images. Distribute the “Golden Ratio in Shapes and Sounds” color sheets. Distribute markers or paint. Ask students to create their own designs.
Golden Ratio Overview

July 12, 2015 by Gary Meisner

What makes a single number so interesting that ancient Greeks, Renaissance artists, a 17th century astronomer and a 21st century novelist all would write about it? It’s a number that goes by many names. This “golden” number, 1.61803399, represented by the Greek letter Phi, is known as the Golden Ratio, Golden Number, Golden Proportion, Golden Mean, Golden Section, Divine Proportion and Divine Section. It was written about by Euclid in “Elements” around 300 B.C., by Luca Pacioli, a contemporary of Leonardo Da Vinci, in “De Divina Proportione” in 1509, by Johannes Kepler around 1600 and by Dan Brown in 2003 in his best selling novel, “The Da Vinci Code.” With the movie release of the “The Da Vinci Code”, the quest to know Phi was brought even more into the mainstream of pop culture. The allure of “The Da Vinci Code” was that it creatively integrated fiction with both fact and myth from art, history, theology and mathematics, leaving the reader never really knowing what was truth and what was not. This site studies this golden number Phi, and its mathematical cousin, the Fibonacci sequence (0, 1, 1, 2, 3, 5, 8, …), both of which have roles in the plot of this murder mystery, and distinguishes between the myth and the math.
Mathematics of the Golden Ratio

This Golden Ratio truly is unique in its mathematical properties and pervasive in its appearance throughout nature. The “mathematically challenged” may be more interested in the appearances of Phi in nature, its application to art, architecture and design, and its potential for insights into the more spiritual aspects of life, but let’s begin with the purest of facts about Phi, which are found in mathematics.

Most everyone learned about the number Pi in school, but relatively few curricula included Phi, perhaps for the very reason that grasping all its manifestations often takes one beyond the academic into the realm of the spiritual just by the simple fact that Phi unveils a unusually frequent constant of design that applies to so many aspects of life. Both Pi and Phi are irrational numbers with an infinite number of digits after the decimal point, as indicated by “…”, the ellipsis.

Where Pi or \( \pi \) (3.14…) is the ratio of the circumference of a circle to its diameter, Phi or \( \Phi \) (1.618 …) is the Golden Ratio that results when a line is divided in one very special and unique way. To illustrate, suppose you were asked to take a string and cut it. There’s any number of places that you could cut it, and each place would result in different ratios for the length of the small piece to the large piece, and of the large piece...
to the entire string. There is one unique point, however, at which the ratio of the large piece to the smaller piece is exactly the same as the ratio of the whole string to the larger piece, and at this point this Golden Ratio of both is 1.618 to 1, or Phi.

\[
\begin{align*}
A &= 1.000 = B + C \\
B &= 0.618 \\
C &= 0.382
\end{align*}
\]

\[
\frac{A}{B} = \frac{B}{C} = 1.618 = \Phi
\]

What makes this so much more than an interesting exercise in mathematics is that this proportion appears throughout creation and extensively in the human face and body. It’s found in the proportions of many other animals, in plants, in the solar system and even in the price and timing movements of stock markets and foreign currency exchange. Its appeal thus ranges from mathematicians to doctors to naturalists to artists to investors to mystics.

Part of the uniqueness of Phi is that it can be derived in many other ways than segmenting a line.

- Phi is the only number whose square is greater than itself by one, expressed \textit{mathematically} as \( \Phi^2 = \Phi + 1 = 2.618 \).
- Phi is also the only number whose reciprocal is less than itself by one, expressed as \( 1/\Phi = \Phi - 1 = 0.618 \).

These two qualities of phi can be expressed algebraically as \( a+1=a^2 \) and \( a-1=1/a \). This can then be rearranged and expressed as \( a^2-a-1=0 \). This is a quadratic equation, the only positive solution of which is:

\[
\Phi = \frac{1+\sqrt{5}}{2} = 1.6180339887\ldots
\]

\[
\Phi = (1 + \sqrt{5})/2 = 1.618033988749894820\ldots
\]
Where 1.618 is represented in upper case as Phi or $\Phi$, its near twin or reciprocal, 0.618, is often represented in lower case as phi or $\phi$. Phi is an irrational number, a number which cannot be expressed as a ratio of two integer numbers.

**The Fibonacci Sequence**

The Fibonacci sequence, also a plot element in “The Da Vinci Code,” provides yet another way to derive Phi mathematically. The series is quite simple. Start with 0 and add 1 to get 1. Then repeat the process of adding each two numbers in the series to determine the next one: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, and so on. This pattern is also found in the diagonals of Pascal's Triangle. The relationship to the Golden Ratio or Phi is found by dividing each number by the one before it. The further you go in the series, the closer the result gets to Phi. For example:

\[
\begin{align*}
1/1 &= 1 \\
2/1 &= 2 \\
3/2 &= 1.5 \\
5/3 &= 1.666 \\
13/8 &= 1.625 \\
21/13 &= 1.615
\end{align*}
\]

If you go further into the series and you'll find that $233/144 = 1.61805$, a very close approximation of Phi, which to ten decimal places is 1.6180339887.

**Geometry of the Golden Ratio**

The Golden Ratio is also found in geometry, appearing in basic constructions of an equilateral triangle, square and pentagon placed inside a circle, as well as in more complex three-dimensional solids such as dodecahedrons, icosahedrons and the basis for the shapes of both Carbon 60 and soccer balls.
Johannes Kepler (1571-1630), discoverer of the true elliptical nature of the orbits of the planets in the solar system described it as such: "Geometry has two great treasures: one is the Theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel."

Nature and Life

There are many other fascinating mathematical relationships and oddities in both Phi and the Fibonacci series that can be explored in more depth, but for now let's now take a step away from the purely mathematical and venture into nature, where Phi and the Fibonacci series manifest themselves pervasively, but not universally. Fibonacci numbers frequently appear in the numbers of petals in a flower and in the spirals of plants. The positions and proportions of the key dimensions of many animals are based on Phi. Examples include the body sections of ants and other insects, the wing dimensions and location of eye-like spots on moths, the spirals of sea shells and the
position of the dorsal fins on porpoises. Even the spirals of human DNA embody phi proportions.

**Perceptions of Beauty**

More intriguing yet is the extensive appearance of Phi throughout the human form, in the face, body, fingers, teeth and even our DNA, and the impact that this has on our perceptions of human beauty. Some would argue that beauty is in the eye of the beholder, but there is evidence to support that what we perceive as beauty in women and men is based on how closely the proportions of facial and body dimensions come to Phi. It seems that Phi is hard-wired into our consciousness as a guide to beauty. For this reason, Phi is applied in both facial plastic surgery and cosmetic dentistry as a guide to achieving the most natural and beautiful results in facial features and appearance.

RETRIEVED FROM: [http://www.goldennumber.net/golden-ratio/](http://www.goldennumber.net/golden-ratio/)
The Golden Ratio in Shapes and Sounds: Essay Organizer

Name________________________

Compare and Contrast Chart Graphic Organizer

<table>
<thead>
<tr>
<th>Item #1</th>
<th>Item #2</th>
<th>Item #3</th>
</tr>
</thead>
</table>

How are they alike?

[Blank lines for notes]

How are they different?

[Blank lines for notes]
The Golden Ratio in Shapes and Sounds

Image of sound vibrations in water
The Golden Ratio in Shapes and Sounds

Image of rose window in a cathedral
The Golden Ratio in Shapes and Sounds

Name_______________________

Image of Extreme close-up for a human eye

Image of Rose Window RETRIEVED FROM:

http://arthemisia.co.vu/

MORE Rose Window Imagery

The South Transept rose window in Chartres Cathedral, made c.1225

https://enthusiastical.wordpress.com/category/rose-window/

Image of close up of a human iris RETRIEVED FROM:
http://www.dimitri.co.uk/nature/close-up-eye-iris-pupil.html
English Language Arts Standards » Literacy Standards » Grade 4

Integration of Knowledge and Ideas

RL.4.7: Make connections between the text of a story or drama and a visual or oral presentation of the text, identifying where each version reflects specific descriptions and directions in the text.

RL.4.9: Compare and contrast the treatment of similar themes and topics (e.g., opposition of good and evil) and patterns of events (e.g., the quest) in stories, myths, and traditional literature from different cultures.

W.4.1: Write informative/explanatory texts to examine a topic and convey ideas and information clearly.

W.4.2a: Introduce a topic clearly and group related information in paragraphs and sections; include formatting (e.g., headings), illustrations, and multimedia when useful to aiding comprehension.

W.4.2b: Develop the topic with facts, definitions, concrete details, quotations, or other information and examples related to the topic.

W.4.2c: Link ideas within categories of information using words and phrases (e.g., another, for example, also, because).

W.4.2d: Use precise language and domain-specific vocabulary to inform about or explain the topic.

W.4.2e: Provide a concluding statement or section related to the information or explanation presented.

Key Ideas and Details

RL.4.1: Refer to details and examples in a text when explaining what the text says explicitly and when drawing inferences from the text.

Integration of Knowledge and Ideas

RL.4.7: Make connections between the text of a story or drama and a visual or oral presentation of the text, identifying where each version reflects
specific descriptions and directions in the text.

Text Types and Purposes

W.4.1: Write informative/explanatory texts to examine a topic and convey ideas and information clearly.

English Language Arts Standards » Literacy Standards » Grade 7

Key Ideas and Details

RL.7.1: Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.

Integration of Knowledge and Ideas

RL.7.9: Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history.

English Language Arts Standards » Literacy Standards » Grades 9-10

Research to Build and Present Knowledge

W.9-10.7: Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

W.9-10.1: Draw evidence from literary or informational texts to support analysis, reflection, and research.
Mathematics » Literacy Standards » Grade 4

Operations and Algebraic Thinking 4.OA

A. Use the four operations with whole numbers to solve problems.

2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Measurement and Data 4.MD

A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

C. Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint...

Geometry 4.G

A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Mathematics » Grade 7

The Number System 7.NS

Geometry 7.G

A. Draw, construct, and describe geometrical figures and describe the relationships between them.

6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
Linear, Quadratic, and Exponential Models ★ F-LE

A. Construct and compare linear, quadratic, and exponential models and solve problems.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
The Human Face and the Golden Ratio

http://www.goldenumber.net/face/

Phi matrix and the Human Face

http://www.phimatrix.com/face-beauty-golden-ratio/

The Iris and the Pupil

The iris is a tissue inside the eye that has a hole in the center called the pupil. The iris contains muscles that allow the pupil to become larger (open up or dilate) and smaller (close up or constrict). The iris regulates the amount of light that enters your eye by adjusting the size of the pupil opening.

In bright light, the iris closes (or constricts) and makes the pupil opening smaller to restrict the amount of light that enters your eye.

![The iris in bright light](image)

In dim light, the iris opens (or dilates) and makes the pupil opening larger to increase the amount of light that enters your eye:

![The iris in dim light](image)
In addition, it is the iris that determines your eye color. People with brown eyes have heavily pigmented irises, while people with blue or lighter-colored eyes have irises with less pigment.

Therefore, people with lighter-colored eyes should wear sunglasses outdoors, especially during the summer. According to Prevent Blindness America, extended exposure to ultraviolet (UV) light (such as sunlight) has been linked with cataracts and macular degeneration.

What are Myopia, Hyperopia, Astigmatism, and Presbyopia?

Myopia (nearsightedness)
With myopia, you can see close-up objects clearly, but distant objects are blurred. Myopia occurs when the eyeball is too long for light rays to focus correctly on your retina. The more myopic you are, the blurrier your vision will be at a distance and objects will have to be closer to you in order to see them clearly.

Hyperopia (farsightedness)
With hyperopia, you can see distant objects clearly, but close-up objects are blurred. Hyperopia occurs when the eyeball is too short for light rays to focus correctly on your retina.

Astigmatism
If you have an astigmatism, the surface of your eye (the cornea) is not smoothly curved; instead, the surface is irregular. Normally, the surface of your cornea is rounded, much like a basketball; with astigmatism, however, your cornea is shaped more like a football (American football, that is). This doesn't allow light rays to focus clearly on a single point on your retina. Astigmatism is usually accompanied by myopia or hyperopia and is usually correctable with eyeglasses.

Presbyopia
Presbyopia makes it difficult to focus on close-up words or images. Most people are between 40 and 50 when they begin to realize that the letters of the telephone book are "too small" or that it's necessary to hold the newspaper further away in order to see clearly. At the same time, the ability to focus on objects that are far away remains intact.
Here is what you see with normal vision:

![Normal Vision](National Eye Institute photo)

Here is what you see if you have a refractive error. The image appears blurred at a distance if you have myopia or appears blurred close-up if you have hyperopia or presbyopia.

![Myopia](National Eye Institute photo)

Your Amazing Eyes!

You carry around a pair of cameras in your head so incredible they can work in bright sunshine or at night. Only 2.5cm in diameter, they can bring you the image of a tiny ant or a twinkling star trillions of kilometres away. They can change focus almost instantly and stay focused even when you’re shaking your head around. These cameras are your eyes...
Eye-Popping Fact ONE: A crucial part of your eyes is as flimsy as a wet tissue!

A fly darts towards your head! Light bounces off the insect and enters your eye's cornea, a clear covering over your eye. The light passes through your pupil, the black circle in the centre of the iris, to the lens. The lens focuses the light onto your retina - a thin but vital lining on the back of your eye that is as flimsy as a wet tissue. Your retina acts like camera film, capturing the picture of the fly. This image is sent to the brain, which instantly tells you to - duck!

Eye-Popping Fact TWO: You blink more than 10,000 times a day!

Your sight is incredibly important, so your body has ways to protect your eyes. Each eye sits on a cushion of fat, surrounded by protective bone. Your eyebrows prevent sweat dripping into your eyes, while eyelashes keep dust and other particles out. The eyelids act as windscreen wipers, spreading tear fluid with every blink to keep your eyes moist and wash away bacteria. You blink more than 10,000 times a day! And if anything gets too close, your eyelids slam shut with amazing speed. How fast does this happen? In the blink of an eye - about 2/5 of a second!
Eye-popping Fact THREE: Your eyes adjust in milliseconds to ANY movement of your head!

You bounce your eyes around all the time. Even when you’re not running or jumping, your head doesn’t stay still. Why isn’t everything a blur when you’re moving? The eyes automatically adjust to the movement of your head with great speed and precision. They’re good at following a moving object, and even better at adjusting to the motion of your head. Test it: Keeping your head still, hold up your hand about 30cm away, and quickly move it back and forth. As fast as your eyes are, your fingers become blurry. Now keep your hand still and move your head back and forth. Amazingly, your fingers stay in focus!
Eye-Popping Fact FOUR: Your eyes see everything upside down and backward!

Your eyes are amazing, but the images they send to your brain are a little quirky - they’re upside down, backward and two-dimensional! Lucky for you, the cameras in your head come with an impressive software package - your brain - that can fix these problems. The brain automatically flips the images from your retinas right side up and combines the images from each eye into a three-dimensional picture.

There is a small area of each retina, called a blind spot, that can’t record what you’re seeing. Your brain makes adjustments for this, too. But sometimes it can be fooled! Check it out - hold the tips of your two index fingers together, about 15cm in front of your eyes. Now separate them slightly and look past them at something in the distance. A floating finger that looks like a sausage appears between your fingers. You just fooled your brain into seeing something that isn’t there!

RETRIEVED FROM: http://www.ngkids.co.uk/science-and-nature/human-eye
In order to begin to understand the potential of Bioarchitecture in terms of health and well-being, it makes sense to explore how the brain and nervous system respond to stimuli from and in our environment. One of our primary senses is seeing, as this is usually the predominant vehicle for our experience of physical space, at least initially.

Perception can be defined as the process of attaining awareness or understanding of the environment by organizing and interpreting sensory information. All perception involves signals in the nervous system. How we perceive the space around us initiates a physiological response which is part of a holistic process which can generate feelings of relaxation, joy and an overall sense of comfort. It can also create stress, discordance and a sense of discomfort, especially if the space is visually discordant.

A very interesting field of study has emerged in recent years called neuroaesthetics, which is an attempt to combine neurological research with aesthetics by investigating the experience of beauty and appreciation of art and other visual stimulus on the level of brain functions and mental states. Researchers in this emergent field combine principles from perceptual psychology, evolutionary biology, and neurological potential and functional brain anatomy in order to address the interpretation and meaning of beauty which may underpin the essence of art and architecture.
It has been said that beauty is in (or perceived through) the eye of the beholder and it is here, with this remarkable organ of perception that we shall begin our journey into the world of seeing. In essence, the eye detects light and converts it into electro-chemical impulses in the brain. It collects projected and reflected light from the surrounding environment, regulates its intensity through the iris diaphragm, and focuses it through a flexible and adjustable assembly of lenses to the cells in the retina at the back of the eye.

The retina contains two major types of light-sensitive ‘photoreceptor’ cells used for vision: the rods and the cones, called that because of their shape. In essence the rods respond to light and dark, as they contain a chemical pigment which is sensitive to low light intensity. The cones are responsible for colour vision, differentiation and depth perception, and are able to receive and respond to different frequencies of light.

When light hits the pigments in the retinal nerve cells (which are considered part of the central nervous system), chemical reactions occur, triggering a cascade of electrical signals which travel along the axons (filamentous fibers) until they reach the ends of the cells, and connect with other chemicals which are stored in little cellular bio containers, called vesicles. At this stage the chemicals in
these sacks are released into the interspaces between the brains nerve cells. This is how the reflected light is transformed into an electrical signal that leaps and hops from one nerve cell to the other distributing itself to many regions of the brain.

The primary distribution pathway for this process of perception is called the optic tract, which resembles a large cable like structure linking the eyes and the brain, via a crossing point called the optic chiasma, which allows for parts of both eyes that attend to the right visual field to be processed in the left visual system in the brain, and vice versa. From here the signals are relayed to a switching station in the brain stem and finally lead to the part of the brain that governs vision, called the occipital lobe.

This brief description of the anatomy of the eye and how the light from an object initiates electrical and chemical signals, creating an image in our awareness is only part of the process of perception.

Overall perception is shaped by learning, memory and expectation, and depends on many complex functions of the nervous system occurring mostly outside of our conscious awareness. For instance, before we recognize an object, our eyes see only bits and pieces of it, in effect presenting contrasting edges and geometric lines of form. There is not enough information available to make a continuous line but the brain does an amazing job of connecting the dots and filling in the blanks.

During this process, other parts of the brain are actively and rapidly seeking to match the information with memorized patterns held in storage, and when there is a resonance the link ignites and we recognize (re-cognize, to know again) what it is we are looking at. Depending on what pattern is chosen, different parts of the brain light up, blood flow increases, proteins are formed and utilised, chemicals are released and distributed and nerve cells fire and signal to one another.
Faces are recognized by one part of the brain (the fusiform face area) and objects by another. Live images of the brain using various scanning apparatus display activity in specific areas corresponding to the interpretation of what is being observed. For example, the prefrontal cortex of the brain is previously known for its roles in the perception of coloured objects, decision making and memory, but recent studies have also linked it to the conscious aesthetic experience because it is activated during tasks such as determining the appeal of visual stimuli. The intensity of the electrical activity in this region, specifically the prefrontal dorsolateral cortex, peaks when we are observing a beautiful work of art or perceiving / experiencing a harmonious space.

What is very interesting and relevant to our discussion is that there is a specific part of the neuronal network of the brain that specializes in perceiving and recognizing buildings, or other large structures or forms that are being viewed from a limited array or perspectives. The location of this part of the brain is below the Para hippocampal place area, which plays an important role in the encoding and recognition of scenes. This part of the brain becomes highly active when we view topographical scene stimuli such as large physical structures, cityscapes and rooms or enclosed spaces.

Recent research indicates that the brain uses larger structures such as buildings to map and define the geometry of the local space. When we first look at a scene we are simultaneously forming a contextual narrative of the space whilst also mapping the primary form in terms of shapes, parameters, boundaries and limits.
So how can the scene being viewed and experienced affect us in terms of our feelings of health and well-being? Is there something about the scene that might be discordant or stressful, in terms of how we relate and respond to it?

Science has discovered that there is a pathway at the base of the brain that connects the visual cortex to the Para hippocampal place area, forming a connection between where the retinal signals are first received and the where they are assembled into the perception of a scene. Along this pathway are nerve cells, which express an increased density of receptors for the morphine-like molecules called endorphins. Research has proven that when people view a natural and beautiful scene or geometrical presentation, the nerve cells in the opiate rich pathway become very active. The resulting biochemical release represents a natural high being gifted to you by your own brain in direct response to the scene being viewed and experienced. So what is in the scene that creates this effect, and is it the objects themselves, or the arrangement of them in space?

Some wonderful work done by researchers in Kyoto University in Japan demonstrates how the geometry of a space, place or scene can suggest a
pleasing picture to our subconscious which creates measurable effects in the brain resulting in feelings of connection, groundedness and health / well-being. The study involved symmetry calculations of the five irregular rock clusters in the Ryoan Ji Zen temple and garden, which are arranged and positioned in a sea of racked gravel. The entire scene was designed to be viewed from the hall of the temple, which is located off centre to the left of the garden. The calculations involved analyzing the axes of symmetry within and between the groups of rock, using a method called medial-axis transformation. When the resulting geometrical pattern was generated and reviewed the researchers made a remarkable discovery.

The axes formed the fractal geometry of a tree, with the trunk passing directly through the primary viewpoint of the temple. The natural branching algorithm extended out and contained each of the rock clusters, in an effortless evolution of visual science. When the process was repeated with random groupings of rocks the pattern failed to materialize. So it would seem that the designers of this garden, priests and meditators, were able to place these points of visual focus in an overall matrix that directly mirrored a fundamental generative natural expression, and once which it would seem our perception is wired to react positively to.

I recall sitting there in the sweet spot of perception in this garden, and feeling totally engrossed in the entire scene. There was a sense and feeling of outer
calm and repose, which successfully covered an intensity of timeless attention that felt at once omnipresent and totally captivating.

Fractal patterns, displaying as self-similar branching forms, are seen at every level and scale in nature, from atomic radii, snowflakes, trees, mountain ranges and galaxies.

It is found in the shape and form of the human body, and expresses in the cascading geometry of nerve cells, alveoli in the lungs, blood vessels and even in the folds of the brain itself.

Fractal geometry often utilizes the infinitely recursive and self-referencing geometry of Phi, also called the Golden Ratio or Sacred Cut. This ratio and its relatives form the basis of bioarchitecture design protocol, providing a natural matrix for the evolution of space and form in direct alignment with the movement most sought by life and living systems.
The timeless graphic of Leonardo’s ideal man is well known and points to a recognition of the inherent proportional harmonic of the human body. It may have been the visual inspiration that prompted the pioneering work of Californian plastic surgeon Dr. Stephen Marquardt who revealed that a phi based proportional precision underpinned the perception of perfect beauty in the human face. He collected photographs of faces of the world deemed to be beautiful and began measuring their dimensions. The golden ratio was everywhere, displaying itself in the relationship between all the elements and features of the face.

‘Beautiful’ people’s mouths were 1.618 (golden ratio expressed as a number) wider than their noses, and the nose itself was 1.618 times wider than the tip of their noses. Before you look in the mirror and start neurotically measuring your face it is worth mentioning that nature likes to approximate, in a playful movement towards aligning all the energies to handle change whilst maintaining a healthy and overall attractive average.

Another very revealing study which is worth mentioning here was carried out by scientists and mathematicians who went on a mission of discovery to a South American forest. They were given permission to cut down a tree and carry out a very detailed survey of the tree, imputing the data into a computer program...
which was able to accurately map the fractal geometry / branching algorithm of the tree. They then chose a larger surrounding area of the forest and proceeded to measure all the trees contained in that area, concentrating primarily on the width of the trunks. What they discovered was that the specifics of the fractal geometry of the single tree that was cut down and measured accurately mirrored the distribution of the differing tree thicknesses in the studied area.

In simple terms the pattern of the entire area was fractally mapped onto one tree, showing that the part contains the information necessary to describe the whole. This amount of context reference is astounding, and points towards the ability of nature and natural forms to store and distribute information effortlessly at any scale.

We cannot say for sure why it is that repeating patterns at different scales are pleasing to the eye (and brain / body). There are many theories, most of which point to the reasonable proposal that fractals are intrinsically satisfying to the human mind. When we view or perceive a scene that consists of fractal structures and arrangements, our mind responds to the complex, repetitive, self-similar increasing-decreasing patterns.

The fractally harmonic context rich visual experience creates a perception process which releases bio chemicals that manifest feelings of health, happiness and well-being. When the mind, which is in essence the process of regulating the flow of energy and information, is freed from the rigid boundaries created by boxed scaling, it can move through the doors of perception at will.
So when we design and create a space that is based on the fractal geometry of nature, nesting shapes, forms and ratios in a physical construct, our senses are activated in a holistic integrated way which our brains and bodies find pleasing. This sensory ease facilitates the generation and experience of feelings that are the foundation of health and happiness. If we extend this principle into other realms of sensory perception, such as the overall shape of the space, the quality of light and use of colour, the texture of the materials, the sounds etc., we can increase and enhance the physiological and psychological effect dramatically.

As new born infants, eager to develop and evolve, we are hard and soft wired to be attracted to the face and breasts of our mother, which are perceived as fractal geometric symmetries. This attraction is absolutely vital to the optimal emergence of consciousness, awareness and intelligence. As an extension of this innate instinct we are naturally drawn to place our attention and focus on shapes that are curved, naturalistic in form and fractal in structure.
When we add to this the fractal potential of time as well as space, we can formulate a generative movement to emergence, where focused ritual and intention are nodal symmetry operations anchoring the whole process in a way that life is particularly attracted to. When amplified further by the addition of others sharing the process the eventual energetic profile of the space becomes very rich in terms of all the fields of information that can embed and dance within the space non-destructively.

There are many examples of structures and art works from the past which incorporated all of these elements in a dance of fractal resonance; from ancient gathering spaces, gothic cathedrals and Indian temples to jewelry, pottery and paintings. Whenever the elements are brought together naturally and in accordance with the fractal forms of life, the result is something that can be described as beautiful and healing. We have observed this potential many times in the homes and living spaces designed using these principles; with people reporting very positive reactions and experiences when they live and move through them.
Bioarchitecture has been defined as the art and science of designing and building spaces which create, support and enhance life and living systems. In this regard it would seem that there is a solid basis of evidence to support this definition, at least in terms of how our minds and bodies respond to the shape, form and integrated patterns of space.

Regardless, we shall continue to help people to touch and manifest their ‘HomeDream’, utilizing all of the skills available to us, developing our own awareness and ability to perceive and play.

RETRIEVED FROM: http://academysacredgeometry.com/articles/science-seeing
Extreme Close Ups of the Human Eye

Born in 1976, Suren Manvelyan started to photograph when he was sixteen and became a professional photographer in 2006. His photographic interests span from Macro to Portraits, Creative photo projects, Landscape, and much more. Suren’s photos have been published in numerous magazines and newspapers in Armenia and worldwide. His project, entitled Your beautiful eyes, is currently the most viewed project of all time on Behance with 2,999,655 views and is the second most ‘appreciated’ project of all time with 70,140 likes.

In parallel to photography, for the past ten years Suren has also enjoyed teaching physics, mathematics, projective geometry and astronomy at the Yerevan Waldorf School. From 1997 to 2011 he served as a scientific researcher at the Institute for Physical Research of National Academy of Sciences.

Suren received his PhD in Theoretical Physics from the Yerevan State University in 2001 where his research focused on Quantum Chaos. He received the President Award of the Republic of Armenia next year for his research work in the field of quantum technologies.

Below you will find a collection of my personal favorites; to see the entire gallery, please visit Behance: https://www.behance.net/gallery/Your-beautiful-eyes/428809
You can also find Suren on Facebook at: https://www.facebook.com/SurenManvelyan
Photograph by Suren Manvelyan
Photograph by Suren Manvelyan
(She sighs)
I suppose if it must be, then it must be.

MOTHER SUPERIOR

MONSIGNOR O’HARA

It must be.

MOTHER SUPERIOR

Very well then.

(Back to Deloris and Eddie)
Welcome to Queen of Angels.

EDDIE

Thank you.

MONSIGNOR O’HARA

(To Eddie)
Come Officer Souther, let us make reparations and there is a tawny port you might enjoy before mass.

(Monsignor O’Hara exits)

EDDIE

(To Deloris)
Behave yourself.

(Eddie exits)

(Beat)

DELORIS

Well.

(Beat)

MOTHER SUPERIOR

Yes.

(Beat)

DELORIS

Okay.

(She hears a small echo, then, loudly and off towards the ceiling-)

OKAY!

(It quickly echoes)
(DELORIS)

Great acoustics!

(Yelling back to the ceiling)

I'm in a church!

(It echoes back)

With a bunch of nuns!

(It echoes back)

Yeah—I like that reverb. You know I'm a singer. Professional. Hey—Mother May I— When this is all over with, could I borrow this space for a few weekends? Fridays, Saturdays...

MOTHER SUPERIOR

Sunday is usually booked.

DELORIS

Hey, I just wanna say thanks for letting me stay here and—and I say this to people all the time—I really dig what you nuns are doing. I love your work. I mean at the end of “The Sound of Music,” when you sisters steal the Nazi's car parts so the singing children can get away. That's good stuff.

MOTHER SUPERIOR

Thank you.

DELORIS

So listen, while I'm here, these are my ground rules. I want three meals a day and I'll need two rooms, one for my clothes, one for my down time.

MOTHER SUPERIOR

And these are my rules. You will stay in your room. When you are not in your room, you will behave as a nun. Do you know how a nun behaves?

DELORIS

Hey, I went to catholic school when I was a kid.

MOTHER SUPERIOR

The benefits of which are quite apparent. Now, you will only come out of your room for meals or prayer—do you pray, child?

DELORIS

Well, one time when I saw Donna Summer, she was wearing a white sequin dress and had a white fur—and I said to myself "Jesus Christ I wish I had that dress." Does that count as prayer?

MOTHER SUPERIOR

No.
Sister Act: the Musical
Raise Your Voice Lyrics

DELORIS:
First rule of singin' -
Get the rafters ringin'!
Toss everything in -
dig down deep inside.
When you've got a song worth hearin'.
There's one thing to do -
just keep your fear from interferin', and let that sucker burst through!

Raise your voice!
Lift it up to heaven!
Raise your voice!
Come on, don’t be shy!
If you feel it, why conceal it?
Let your soul rejoice!
Raise the stakes!
Raise your game!
Raise your voice!
The Physics of Singing

Entry posted by leahmaew · 24 Apr 2013

A lot of people wouldn't know that singing actually involves a great deal of physics. For example the carrying of sound from a singer to the listener, has to do with mechanical waves. Different people have different types of voices for singing, the most common difference in peoples’ singing voices is their range of voice. Voice ranges can reach very high notes, or they can reach very low notes. A good singer is most likely able to hit very low notes as well as very high notes. The pitch, or the high or lowness that someone can sing, is associated with frequency.

Sound is a mechanical wave because it requires a medium to travel, when someone sings, the higher frequency they sing in, the higher their pitch will be, for example, someone with a squeaky voice like Taylor Swift sings at a high frequency, while someone with a low raspy voice like Adele sings at a lower frequency. Frequency is measured in hertz and the frequency of a wave refers to how often the particles of the medium vibrate when a wave passes through the medium. Frequency is measured as the number of wave cycles that occur in one second. 1 hertz = 1 vibration/second

Amplitude of a sound wave is also part of singing. When someone sings with a greater amplitude compared to a smaller amplitude, they are singing louder compared to singing quieter. Amplitude is the size of the vibration in a sound wave that affects how loud we sing. Larger vibrations make a louder sound.

Diffraction is also used in singing because it helps us to explain why sound can be heard from different rooms when someone is singing. Diffraction uses the edges of barriers to send sound waves out in a different direction than they were originally going. For example if someone was performing a solo in a high school auditorium and someone opened the doors, now most likely the whole hallway would be able to hear them. That is because with long wavelengths and small openings, sound diffractions more and therefore will be sent out further. Singing involves a lot of physics and without understanding how sound waves travel or how to magnify voices then one would not be able to master the art of singing.

**Wavelength** is represented with the Greek letter lambda: \( \lambda \). It is equal to the velocity of the wave, divided by the frequency. **Wavelength** is expressed in units of meters (m).

\[
\lambda = \frac{v}{f}
\]

*Where,*

\( \lambda \) = Wavelength  
\( v \) = Velocity of propagation  
\( f \) = Frequency of signal

The lower-case Greek letter “lambda” (\( \lambda \)) represents wavelength, in whatever unit of length used in the velocity figure (if miles per second, then wavelength in miles; if meters per second, then wavelength in meters). Velocity of propagation is usually the speed of light when calculating signal wavelength in open air or in a vacuum, but will be less if the transmission line has a velocity factor less than 1.

**Wavelength Formula**  
**Wavelength** is defined as the difference between two successive crust or trough. It is denoted by \( \lambda \).

**Wavelength Formula** for any wave given by

\[
\lambda = \frac{v}{f}
\]

*Where,* \( v \) is the wave speed and \( f \) is the frequency taken.

**The Formula for wavelength** for light wave is given by

\[
\lambda = \frac{c}{f}
\]

*Where,* \( c \) is the velocity of light, \( f \) is the frequency of the wave.

The **wave speed formula** is given by
Wavelength is expressed in \( \text{m} \), velocity of wave is expressed in \( \text{m/s} \), frequency is expressed in \( \text{Hz} \). Wavelength formula is used to find wavelength or frequency if any of these parameters are given.

### What is a Medium?

But what is meant by the word *medium*? A **medium** is a substance or material that carries the wave.

#### Particle-to-Particle Interaction

To fully understand the nature of a wave, it is important to consider the medium as a collection of interacting *particles*. In other words, the medium is composed of parts that are capable of interacting with each other. The interactions of one particle of the medium with the next adjacent particle allow the disturbance to travel through the medium.

For a wave to be transmitted through a medium, the individual particles of the medium must be able to interact so that they can exert a push and/or pull on each other; this is the mechanism by which disturbances are transmitted through a medium.

Combining this information with the equation for speed (speed = distance/time), it can be said that the speed of a wave is also the wavelength/period.

\[
\text{Speed} = \frac{\text{Wavelength}}{\text{Period}}
\]

Since the period is the reciprocal of the frequency, the expression \( 1/f \) can be substituted into the above equation for period. Rearranging the equation yields a new equation of the form:

\[
\text{Speed} = \text{Wavelength} \cdot \text{Frequency}
\]

The above equation is known as the wave equation. It states the mathematical relationship between the speed (\( v \)) of a wave and its wavelength (\( \lambda \)) and frequency (\( f \)). Using the symbols \( v \), \( \lambda \), and \( f \), the equation can be rewritten as

\[
v = f \cdot \lambda
\]
Diffraction is also used in singing because it helps us to explain why sound can be heard from different rooms when someone is singing. Diffraction uses the edges of barriers to send sound waves out in a different direction than they were originally going. For example if someone was performing a solo in a high school auditorium and someone opened the doors, now most likely the whole hallway would be able to hear them. That is because with long wavelengths and small openings, sound diffractions more and therefore will be sent out further. Singing involves a lot of physics and without understanding how sound waves travel or how to magnify voices then one would not be able to master the art of singing.
One of the more well-known songs from the musical Sister Act is Raise Your Voice. The lyrics begin: *First rule of singin’ - Get the rafters ringin’!* This lesson gives students the opportunity to raise their own voices, learn about the physics behind resonance, and compare the sound of their voices with the rest of the class.

Begin by sharing the definitions of resonance and sine waves.

**res-o-nance  rezənəns/**

1. 1. the quality in a sound of being deep, full, and reverberating.
   "the resonance of his voice"

2. 2. PHYSICS
   the reinforcement or prolongation of sound by reflection from a surface or by the synchronous vibration of a neighboring object.

**Introducing the Sine Wave (Pure Tone)**

The most fundamental sound is the sine wave, characterized by a single frequency without any harmonics. Sine waves can be easily recognized by ear, as they sound clear and pure.

*In 1822, French mathematician Joseph Fourier discovered that any waveform could be broken up as a combination of sine waves with different amplitude and frequencies. The discipline of reducing a complex waveform to a combination of sine waves is called Fourier analysis, and is fundamental to sound processing.*

As sine waves are made up by a single frequency, they are best suited to test audio systems at a particular frequency.
Next, share scientific information about how humans sing. Display the information where everyone can see it, such as a dry-erase board, Promethean board or ELMO.
Vocal Sound Production

Diaphragm action pushes air from the lungs through the vocal folds, producing a periodic train of air pulses. This pulse train is shaped by the resonances of the vocal tract. The basic resonances, called vocal formants, can be changed by the action of the articulators to produce distinguishable voice sounds, like the vowel sounds.

The Vocal Folds

Positioned at the base of the larynx in the vocal tract, these twin infoldings of mucous membrane act as the vibrator or "reed" during phonation. Open during breathing, the folds are closed by the pivoting of the arytenoid cartilages for speech or singing.
Positive air pressure from the lungs forces them open momentarily, but the high velocity air produces a lowered pressure by the Bernoulli effect which brings them back together. The folds themselves have a resonant frequency which determines voice pitch.

In an adult male, the vocal folds are usually 17-23 mm long, and 12.5 -17 mm in an adult female (Kaplan). They may be stretched 3 or 4 mm by action of the muscles in the larynx.

The male speaking voice averages about 125 Hz, while the female voice averages about 210 Hz. Children's voices average over 300 Hz. The illustration below shows approximate pitches for speaking voices related to an equal tempered piano keyboard based on $A_4 = 440$ Hz.

**Vocal Folds in Phonation**

The process of converting the air pressure from the lungs into audible vibrations is called phonation. When the air passes through the elastic vocal folds and causes them to vibrate, the type of phonation is called voicing. The vocal folds give the singer a wide range of control over the pitch of the sound produced. While "vocal folds" is more descriptive than "vocal cords", there is some similarity to a vibrating string in that the pitch produced depends upon the length, mass and tension of the vocal folds.
The **excitation** of the vocal folds is however very different from the excitation of a string in that it is caused by the passage of air through the opening between the folds. The muscles of the larynx change the elasticity and tension of the vocal folds to determine the pitch of the sound.

**Vocal Fold Excitation**

The vibratory cycle of the vocal folds is driven by aerodynamic phenomena. Driving air pressure from the lungs controls the opening of the folds, and the Bernoulli effect controls the closing phase. As the top of the folds is opening, the bottom is in the process of closing, and as soon as the top is closed, the pressure buildup begins to open the bottom. The vibration is then like a wave which travels from the bottom to the top of the vocal folds. Each vibration allows a brief puff of air to escape, producing an audible sound at the frequency of the opening; this process is called voicing.

The voice intensity can be increased by increasing the air flow from the lungs and increasing the resistance from the vocal folds. The vocal folds are blown wider apart, and stay apart longer during the cycle. This increases the amplitude of the sound pressure wave produced. How about this for a "big word" description of the process:

**myoelastic aerodynamic theory of phonation**
Next, explain that students are going to have an opportunity to learn about their own voices. Everyone can hold one note, or a pitch. This exercise will help the class explore the average vocal range of the students and demonstrate how physics principals and mathematics can be applied to signing.

Begin by passing out a class pitch table. Next, using the internet, open a search engine and go to this on-line tone generator: http://plasticity.szynalski.com/tone-generator.htm One at a time, have each student choose a note they can sustain, and then sustain it for 30 seconds while the Hz is located. Once the pitch is located, ask students to record the student’s information (name and Hz) on the class pitch table. Once everyone has had a chance to sign and sustain a note, distribute the “Keyboard and frequencies” chart. Ask students to find the piano pitch for the Hz and record it on the table; many times this will have to be done as an approximation. As an example, we conducted this same exercise in the JPAS office. Here are our results

<table>
<thead>
<tr>
<th>People in the office</th>
<th>Tone</th>
<th>Piano Pitch, Approximate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theo</td>
<td>652 Hz</td>
<td>E5</td>
</tr>
<tr>
<td>Karel</td>
<td>693 Hz</td>
<td>F5</td>
</tr>
<tr>
<td>Aaron</td>
<td>678 Hz</td>
<td>E5</td>
</tr>
<tr>
<td>Joshua</td>
<td>227 Hz</td>
<td>A3/B3</td>
</tr>
<tr>
<td>Donna</td>
<td>245 Hz</td>
<td>B3</td>
</tr>
<tr>
<td>Nick</td>
<td>245 Hz</td>
<td>B3</td>
</tr>
<tr>
<td>George</td>
<td>262 Hz</td>
<td>C4 (middle C)</td>
</tr>
</tbody>
</table>

Now ask students to compare the different frequencies (Hz) within the class—the pitch each person sang in the class. Distribute the “Circle Graph” and “Box and Whisker Plot” sheets. Ask students to use their Use the “Class Pitch Table” to complete the graphs. As a class, discuss the student’s findings.
# Raise Your Voice Pitch Table

<table>
<thead>
<tr>
<th>Student Name</th>
<th>Tone (Hz)</th>
<th>Piano Pitch, Approximate</th>
<th>Vocal Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Name</td>
<td>Tone (Hz)</td>
<td>Piano Pitch, Approximate</td>
<td>Vocal Range</td>
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</tbody>
</table>
• Keyboard and frequencies •
• Naming of musical notes and piano keys •

Music notation systems – Music frequencies of equal temperament tuning

Chord name finder by note entry

The English and American scientific system versus the German system
Scientific Pitch Notation (SPN), also known as American Standard Pitch Notation

The standard pitch tuning is A4 (A above middle C) – in German-speaking countries it is called a’.

The notes have different names. The German system is used also in many other countries, as there are e.g. Poland, Czech Republic, and Russia.

A "normal" piano has the following dimensions: width 145 to 150 cm, height 115 to 125 cm, depth 52 to 60 cm.

A typical "keyboard" has 61 keys today. The sound frequencies of the 88 keys on the piano are:

<table>
<thead>
<tr>
<th>Piano key number</th>
<th>English notation</th>
<th>German notation</th>
<th>Frequency Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>88</td>
<td>C8 - Last tone</td>
<td>c&quot;&quot;&quot;&quot; - Highest</td>
<td>4186.01</td>
</tr>
<tr>
<td>87</td>
<td>B7</td>
<td>h&quot;&quot;&quot;&quot;</td>
<td>3951.07</td>
</tr>
<tr>
<td>86</td>
<td>A$7/Bl7</td>
<td>ais&quot;&quot;&quot;&quot;/b&quot;&quot;&quot;&quot;</td>
<td>3729.31</td>
</tr>
<tr>
<td>85</td>
<td>A7</td>
<td>a&quot;&quot;&quot;&quot;</td>
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<td>Ais/B</td>
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<td>G$\sharp$/Al$2$</td>
<td>Gis/As</td>
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<td>G2</td>
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<td>F$\sharp$/Gl$2$</td>
<td>Fis/Ges</td>
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<td>D$\sharp$/El$2$</td>
<td>Dis/Es</td>
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<td>D</td>
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<td>A$\sharp$/Bl$1$</td>
<td>,Ais/,,B ~60 Hz</td>
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<td>13</td>
<td>A1</td>
<td>,A</td>
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<td>12</td>
<td>G$\sharp$/Al$1$</td>
<td>,Gis/,,As</td>
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<td>11</td>
<td>G1</td>
<td>,G ,~50 Hz</td>
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<td>,E</td>
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<td>D$\sharp$/El$1$</td>
<td>,Dis/,,Es</td>
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<td>,D</td>
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<td>,Cis/,,Des</td>
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<td>2</td>
<td>A$\flat$/Bl$0$</td>
<td>,,Ais/,,B</td>
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<td>1</td>
<td>A0 - First tone</td>
<td>,,A - Lowest</td>
<td>27.5000</td>
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</table>

RETRIEVED FROM:  http://www.sengpielaudio.com/calculator-notenames.htm
Raise Your Voice

Name_______________________

Compare the different frequency (Hz) each person sang in the class over a 30 second interval. Create a circle graph to compare percentages: how many people had a pitch of at least A) 227 Hz, B) 280 Hz, C) 350Hz, D) 580Hz or E) above 750Hz?
Raise Your Voice

Name_______________________

Use the “Class Pitch Table” to create a box-and-whisker plot to compare the different pitches of each person in the class.

Arrange all the pitches from lowest to highest and find the median ($Q_2$ or Med on the calculator.) The median is the number exactly in the middle of this ordered set of numbers. Now look at the numbers on the left side of the median. Find the number exactly in the middle of this set of numbers. This number is called the lower quartile ($Q_1$ on the calculator.) Now look at the numbers on the right side of the median. Find the number exactly in the middle of this set of numbers. This number is called the upper quartile ($Q_3$ on the calculator.) Now subtract the lower quartile from the upper quartile. This number is the interquartile range (IQR).

Box-and-Whisker Plot
Resonance definition RETRIEVED FROM: https://www.google.com/webhp?sourceid=chrome-instant&ion=1&espv=2&ie=UTF-8&q=what%20is%20resonance

Sine Wave definition RETRIEVED FROM: http://www.audiocheck.net/audiofrequencysignalgenerator_sinetone.php

Information about vocal folds and myoelastic aerodynamic theory of phonation RETRIEVED FROM: http://hyperphysics.phy-astr.gsu.edu/hbase/music/voice.html
Mathematics Standards » Literacy Standards » Grade 4

Operations and Algebraic Thinking 4.OA

A. Use the four operations with whole numbers to solve problems.

Measurement and Data 4.MD

A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line

Mathematics » Grade 7

The Number System 7.NS

d. Apply properties of operations as strategies to add and subtract rational numbers.

Expressions and Equations 7.EE

B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Mathematics Standards » Algebra II

Linear, Quadratic, and Exponential Models★ F-LE
A. Construct and compare linear, quadratic, and exponential models and solve problems.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

B. Interpret expressions for functions in terms of the situation they model.
ADDITIONAL RESOURCES

Stained Glass
http://academysacredgeometry.com/articles/science-seeing

The physics of Singing
http://discovermagazine.com/1999/aug/physics
http://physicsbuzz.physicscentral.com/2012/12/the-science-of-hitting-high-notes.html
http://hyperphysics.phy-astr.gsu.edu/hbase/music/voice.html
http://hendrix2.uoregon.edu/~dlivelyb/phys152/L10.html
http://www.phy.mtu.edu/~suits/scales.html
http://www.physicsclassroom.com/class/sound/Lesson-4/Fundamental-Frequency-and-Harmonics
http://www.sengpielaudio.com/calculator-harmonics.htm
https://www.dmt-nexus.me/forum/default.aspx?g=posts&t=24990

The Mathematics of Harmony:
http://www.amazon.com/exec/obidos/ASIN/981277582X/phipoint-20
http://www.sengpielaudio.com/calculator-wavelength.htm
http://cymatics.ning.com/profile/JodinaMeehan
http://www.mushroom-magazine.com/cymatics-invisible-dance-frequency/
http://nigelstanford.com/Cymatics/Speaker_Dish